

# Math 109

# Practice Final Exam

Print Name: \_\_\_\_\_

Section: \_\_\_\_\_

## Statement of Ethics regarding this exam

I agree to complete this exam without unauthorized assistance from any person, materials, or device.

Signature: \_\_\_\_\_

Date: \_\_\_\_\_

- This is a 3 hour **closed book** exam. **No notes, books, or calculators are allowed.**
- Present your solution to each problem in a clear and orderly fashion. **Show all your work.** An answer without justification will receive little to no credit.

The table on the right is for grading purposes. Please do not write in it.

Question	Points	Score
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
7	20	
8	20	
9	20	
10	20	
11	20	
12	15	
Total:	235	

1. (20 points) Compute the following integral

$$\int_{1/2}^1 \frac{e^{1/x}}{x^3} dx$$

2. (20 points) Compute the following integral

$$\int \sin(x) \tan^2(x) dx$$

3. (20 points) Compute the following integral

$$\int \frac{x^3}{\sqrt{9+x^2}} dx$$

4. (20 points) Solve the following differential equation

$$\frac{dy}{dx} + y \tan x = \sec x; y(0) = 0$$

5. (20 points) Solve the following differential equation

$$\frac{dy}{dx} = -\frac{y(1 + 2x^2)}{x}; y(1) = 1$$

6. (20 points) Consider the parametric curve

$$x(t) = t^3 + t^2 + t - 1; \quad y(t) = t^2 + t - 1.$$

Find the intersection of this curve with the line  $y = x$ . What is the slope of the curve at that point?

7. (20 points) Find the area enclosed by the curve

$$x = t - \frac{1}{t}; y = t + \frac{1}{t}; t > 0$$

and the line  $y = 2.5$ .



8. (20 points) Sketch the polar curves  $r = 2$  and  $r = 2 - \cos \theta$ . Identify the region consisting of points enclosed by both curves. Find the area of this region.

9. (20 points) Find the interval of convergence of the following power series. Make sure you justify your answer.

$$\sum_{n=1}^{\infty} \sin\left(\frac{1}{n}\right)(x-1)^n$$

10. For the following series, determine whether they converge or diverge. You need to justify your answer

(a) (10 points)  $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$

(b) (10 points)  $\sum_{n=1}^{\infty} \frac{e^{-n}}{n^3}$

11. (20 points) Use the degree 8 Maclaurin polynomial of  $\sin(x^2)$  to approximate the integral

$$\int_0^{0.1} \sin(x^2) dx.$$

What is the upper bound of the error of this estimate according to the alternating series test? (You do not need to simplify your answer.)

12. You do not have to justify your answers to this question.

(a) (5 points) Give an example of a parametric curve  $x = f(t)$ ;  $y = g(t)$ ;  $0 \leq t \leq 1$  which is NOT a graph of a function. (A curve is a graph of a function if it can be specified by  $y = h(x)$ ;  $a \leq x \leq b$  for some function  $h$ ).

(b) (5 points) Give an example of a first order differential equation that is neither linear nor separable.

(c) (5 points) Give an example of a conditionally convergent series.

## Trigonometric Identities

- Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \csc^2 x$$

- Sum and Difference Formulas

$$\sin(A \pm B) = \sin A \cos B \pm \sin B \cos A$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

- Double Angle Formulas

$$\sin(2A) = 2 \sin A \cos A$$

$$\cos(2A) = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$$

$$\tan(2A) = \frac{2 \tan A}{1 - \tan^2 A}$$

- Half Angle Formulas

$$\sin^2 x = \frac{1 - \cos(2x)}{2}$$

$$\cos^2 x = \frac{1 + \cos(2x)}{2}$$

$$\tan^2 x = \frac{1 - \cos(2x)}{1 + \cos(2x)}$$

- Product Formulas

$$\sin A \sin B = \frac{1}{2} (\cos(A - B) - \cos(A + B))$$

$$\cos A \cos B = \frac{1}{2} (\cos(A - B) + \cos(A + B))$$

$$\sin A \cos B = \frac{1}{2} (\sin(A + B) + \sin(A - B))$$

## Differentiation Formulas

$$\frac{d}{dx}(\ln |x|) = \frac{1}{x}$$

$$\frac{d}{dx}(a^x) = a^x \ln a$$

$$\frac{d}{dx}(\sin x) = \cos x$$

$$\frac{d}{dx}(\cos x) = -\sin x$$

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\frac{d}{dx}(\cot x) = -\csc^2 x$$

$$\frac{d}{dx}(\sec x) = \sec x \tan x$$

$$\frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$$

$$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{x^2+1}$$

$$\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$$

$$\frac{d}{dx}(\csc^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$$

## Integration Formulas

$$\int \frac{1}{x} dx = \ln |x| + C$$

$$\int a^x dx = \frac{1}{\ln a} a^x + C$$

$$\int \ln x dx = x \ln x - x + C$$

$$\int \sin x dx = -\cos x + C$$

$$\int \cos x dx = \sin x + C$$

$$\int \tan x dx = \ln |\sec x| + C$$

$$\int \cot x dx = -\ln |\csc x| + C$$

$$\int \sec x dx = \ln |\sec x + \tan x| + C$$

$$\int \csc x dx = -\ln |\csc x + \cot x| + C$$

$$\int \sec^2 x dx = \tan x + C$$

$$\int \csc^2 x dx = -\cot x + C$$

$$\int \sec x \tan x dx = \sec x + C$$

$$\int \csc x \cot x dx = -\csc x + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1} \frac{|x|}{a} + C$$

## Trigonometric Substitution

$$\sqrt{a^2 - x^2} \implies x = a \sin \theta$$

$$\sqrt{a^2 + x^2} \implies x = a \tan \theta$$

$$\sqrt{x^2 - a^2} \implies x = a \sec \theta$$